

The background features abstract, colorful swirls in shades of purple, green, and blue, interspersed with small yellow triangles. The text is centered in the lower half of the image.

# ***t*-test**

**Mahmoud Alhussami, DSc., Ph.D.**



# Learning Objectives

- Compute by hand and interpret
  - Single sample  $t$
  - Independent samples  $t$
  - Dependent samples  $t$
- Use SPSS to compute the same tests and interpret the output



# Review 6 Steps for Significance Testing

1. Set alpha ( $p$  level).
2. State hypotheses, Null and Alternative.
3. Calculate the test statistic (sample value).
4. Find the critical value of the statistic.
5. State the decision rule.
6. State the conclusion.



# $t$ -test

- $t$ -test is about means: distribution and evaluation for group distribution
- Withdrawn from the normal distribution
- The shape of distribution depends on sample size and, the sum of all distributions is a normal distribution
- $t$ -distribution is based on sample size and varies according to the degrees of freedom



# What is the $t$ -test

- $t$  test is a useful technique for comparing mean values of two sets of numbers.
- The comparison will provide you with a statistic for evaluating whether the difference between two means is statistically significant.
- $t$  test is named after its inventor, William Gosset, who published under the pseudonym of student.
- $t$  test can be used either :
  - 1.to compare two independent groups (independent-samples  $t$  test)
  - 2.to compare observations from two measurement occasions for the same group (paired-samples  $t$  test).



# What is the t -test

- The null hypothesis states that any difference between the two means is a result to difference in distribution.
- Remember, both samples drawn randomly form the same population.
- Comparing the chance of having difference is one group due to difference in distribution.
- ***Assuming that both distributions came from the same population, both distribution has to be equal.***



# What is the $t$ -test

- Then, what we intend:

**“To find the difference due to chance”**

- Logically, The larger the difference in means, the more likely to find a significant  $t$  test.

- But, recall:

1. Variability

More variability = less overlap = larger difference

2. Sample size

Larger sample size = less variability (pop) = larger difference



# Types

1. The **one-sample  $t$  test** is used to compare a single sample with a population value. For example, a test could be conducted to compare the average salary of GP within a hospital with a value that was known to represent the national average for physicians.
2. The **independent-sample  $t$  test** is used to compare two groups' scores on the same variable. For example, it could be used to compare the salaries of dentists and physicians to evaluate whether there is a difference in their salaries.
3. The **paired-sample  $t$  test** is used to compare the means of two variables within a single group. For example, it could be used to see if there is a statistically significant difference between starting salaries and current salaries among the general physicians in an organization.





# Assumption

1. Dependent variable should be continuous (I/R)
2. The groups should be randomly drawn from normally distributed and independent populations

e.g. Male X Female  
Dentist X Physician  
Manager X Staff


**NO OVER LAP**



# Assumption

3. the independent variable is categorical with two levels
4. Distribution for the **two independent** variables is normal
5. Equal variance (homogeneity of variance)
6. large variation = less likely to have sig t test = accepting null hypothesis (fail to reject) = Type II error = a threat to power

Sending an innocent to jail for no significant reason



# Story of power and sample size

- Power is the probability of rejecting the null hypothesis
- The larger the sample size is most probability to be closer to population distribution
- Therefore, the sample and population distribution will have less variation
- Less variation the more likely to reject the null hypothesis
- **So, larger sample size = more power = significant t test**



# One Sample Exercise (1)

Testing whether light bulbs have a life of 1000 hours

1. Set alpha.  $\alpha = .05$
2. State hypotheses.
  - Null hypothesis is  $H_0: \mu = 1000$ .
  - Alternative hypothesis is  $H_1: \mu \neq 1000$ .
3. Calculate the test statistic

# Calculating the Single Sample t

800
750
940
970
790
980
820
760
1000
860

What is the mean of our sample?

$$\bar{X} = 867$$

What is the standard deviation for our sample of light bulbs?

$$SD = 96.73$$

$$SE = \frac{SD}{\sqrt{N}} = \frac{96.73}{\sqrt{10}} = 30.59$$

$$t_{\bar{X}} = \frac{\bar{X} - \mu}{S_{\bar{X}}} = \frac{867 - 1000}{30.59} = -4.35$$



# Determining Significance

4. Determine the critical value. Look up in the table (Munro, p. 451). Looking for  $\alpha = .05$ , two tails with  $df = 10 - 1 = 9$ . Table says 2.262.
5. State decision rule. If absolute value of sample is greater than critical value, reject null.  
If  $|-4.35| > |2.262|$ , reject  $H_0$ .



# Finding Critical Values

A portion of the t distribution table

	Proportion in one tail					
	0.25	0.10	0.05	0.025	0.01	0.005
<i>df</i>	Proportion in two tails combined					
	0.50	0.20	0.10	0.05	0.02	0.01
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707

Table A.3 Values for Student's *t* Distribution

df	Two-tailed test		
	.10	.05	.01
1	6.314	12.706	63.657
2	2.920	4.302	9.925
3	2.353	3.182	5.841
4	2.132	2.776	4.604
5	2.015	2.571	4.032
6	1.943	2.447	3.707
7	1.895	2.365	3.499
8	1.860	2.306	3.355
9	1.833	2.262	3.250
10	1.812	2.228	3.169
11	1.796	2.201	3.106
12	1.782	2.179	3.055
13	1.771	2.160	3.012
14	1.761	2.145	2.977
15	1.753	2.131	2.947
16	1.746	2.120	2.921
17	1.740	2.110	2.898
18	1.734	2.101	2.878
19	1.729	2.093	2.861
20	1.725	2.086	2.845

# t Values

- Critical value decreases if N is increased.
- Critical value increases if alpha is increased.
- Differences between the means will not have to be as large to find sig if N is large or alpha is increased.





# Stating the Conclusion

6. State the conclusion. We reject the null hypothesis that the bulbs were drawn from a population in which the average life is 1000 hrs. The difference between our sample mean (867) and the mean of the population (1000) is SO different that it is unlikely that our sample could have been drawn from a population with an average life of 1000 hours.



# SPSS Results

## One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
BULBLIFE	10	867.0000	96.7299	30.5887

## One-Sample Test

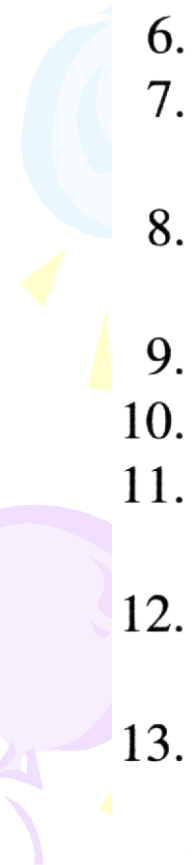
	Test Value = 1000					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
BULBLIFE	-4.348	9	.002	-133.0000	-202.1964	-63.8036



Computers print  $p$  values rather than critical values. If  $p$  (Sig.) is less than .05, it's significant.



# Steps For Comparing Groups

1. Define the population of interest.
  2. State the problem.
  3. Review the literature. Determine if the problem is solved by prior research.
  4. If the problem is not solved, state an hypothesis ( $H_0$  = null,  $H_1$  = directional).
  5. Select a level of confidence (consider consequences of error).
  6. Select a power level and determine appropriate sample size.
  7. Randomly select two samples from the population. Treat one sample with the independent variable. Provide appropriate controls for the other sample.
  8. Compute mean, standard deviation ( $N - 1$ ), and standard error of the mean for each sample.
  9. Compute standard error of the difference and  $t$ .
  10. Determine degrees of freedom.
  11. Compare obtained  $t$  to critical values in a table of  $t$  or read  $p$  value from a computer.
  12. Make a conclusion by accepting or rejecting the hypothesis at the given level of confidence.
  13. Determine the practical importance of the conclusion by calculating the size of the effect.
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The background features several large, flowing, abstract shapes in light green, light purple, and light blue. Scattered throughout are numerous small, yellow, triangular shapes, some pointing upwards and others downwards, creating a festive or celebratory feel.

# **t-tests with Two Samples**

**Independent  
Samples t-test**

**Dependent  
Samples t-test**

# Independent Samples *t*-test

- Used when we have two independent samples, e.g., treatment and control groups.

- Formula is: 
$$t_{\bar{X}_1 - \bar{X}_2} = \frac{\bar{X}_1 - \bar{X}_2}{SE_{diff}}$$

- Terms in the numerator are the sample means.
- Term in the denominator is the standard error of the difference between means.



# Independent samples *t*-test

The formula for the standard error of the difference in means:

$$SE_{diff} = \sqrt{\frac{SD_1^2}{N_1} + \frac{SD_2^2}{N_2}}$$

Suppose we study the effect of caffeine on a motor test where the task is to keep a the mouse centered on a moving dot. Everyone gets a drink; half get caffeine, half get placebo; nobody knows who got what.

# Independent Sample Data

(Data are time off task)

Experimental (Caff)	Control (No Caffeine)
12	21
14	18
10	14
8	20
16	11
5	19
3	8
9	12
11	13
	15
$N_1=9, M_1=9.778, SD_1=4.1164$	$N_2=10, M_2=15.1, SD_2=4.2805$



# Independent Sample Steps(1)

1. Set alpha. Alpha = .05
2. State Hypotheses.

Null is  $H_0: \mu_1 = \mu_2$ .

Alternative is  $H_1: \mu_1 \neq \mu_2$ .



# Independent Sample Steps(2)

3. Calculate test statistic:

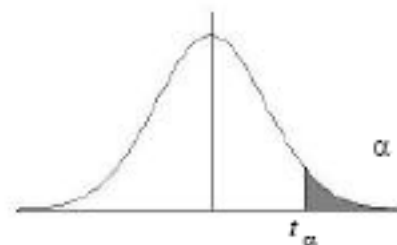
$$t = \frac{\bar{X}_1 - \bar{X}_2}{SE_{diff}} = \frac{9.778 - 15.1}{1.93} = \frac{-5.322}{1.93} = -2.758$$

$$SE_{diff} = \sqrt{\frac{SD_1^2}{N_1} + \frac{SD_2^2}{N_2}} = \sqrt{\frac{(4.1164)^2}{9} + \frac{(4.2805)^2}{10}} = 1.93$$

# Independent Sample Steps (3)

4. Determine the critical value. Alpha is .05, 2 tails, and  $df = N1 + N2 - 2$  or  $10 + 9 - 2 = 17$ . The value is 2.11.
5. State decision rule. If  $|-2.758| > 2.11$ , then reject the null.
6. Conclusion: Reject the null. the population means are different. Caffeine has an effect on the motor pursuit task.

Table 4: Percentage Points of the  $t$  distribution



df	$\alpha$					
	0.250	0.100	0.050	0.025	0.010	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
⋮						
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
120	0.677	1.289	1.658	1.980	2.358	2.617
$\infty$	0.674	1.282	1.645	1.960	2.326	2.576



# Using SPSS

- Open SPSS
- Open file “SPSS Examples” for Lab 5
- Go to:
  - “Analyze” then “Compare Means”
  - Choose “Independent samples t-test”
  - Put IV in “grouping variable” and DV in “test variable” box.
  - Define grouping variable numbers.
    - E.g., we labeled the experimental group as “1” in our data set and the control group as “2”

# Independent Samples Exercise

Experimental	Control
12	20
14	18
10	14
8	20
16	

Work this problem by hand and with SPSS.  
You will have to enter the data into SPSS.

# SPSS Results

## Group Statistics

GROUP		N	Mean	Std. Deviation	Std. Error Mean
TIME	experimental group	5	12.0000	3.1623	1.4142
	control group	4	18.0000	2.8284	1.4142

## Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means					
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference
									Lower Upper
TIME	Equal variances assumed	.130	.729	-2.958	7	.021	-6.0000	2.0284	-10.7963 -1.2037
	Equal variances not assumed			-3.000	6.857	.020	-6.0000	2.0000	-10.7493 -1.2507

The background features several large, stylized, overlapping swirls in light green, light blue, and light purple. Scattered throughout are numerous small, yellow, starburst-like shapes, some of which are larger and more prominent than others. The overall aesthetic is clean and modern.

# **Dependent Samples t-tests**



# Dependent Samples *t*-test

- Used when we have dependent samples –
  - matched, paired or tied somehow
    - Repeated measures
    - Brother & sister, husband & wife
    - Left hand, right hand, etc.
- Useful to control individual differences. Can result in more powerful test than independent samples *t*-test.





# Dependent Samples $t$

Formulas:

$$t_{\bar{X}_D} = \frac{\bar{D}}{SE_{diff}}$$

$t$  is the difference in means over a standard error.

$$SE_{diff} = \frac{SD_D}{\sqrt{n_{pairs}}}$$

The standard error is found by finding the difference between each pair of observations. The standard deviation of these difference is  $SD_D$ . Divide  $SD_D$  by sqrt (number of pairs) to get  $SE_{diff}$ .

# Another way to write the formula

$$t_{\bar{X}_D} = \frac{\bar{D}}{SD_D / \sqrt{n_{pairs}}}$$



# Dependent Samples $t$ example

Person	Painfree (time in sec)	Placebo	Difference
1	60	55	5
2	35	20	15
3	70	60	10
4	50	45	5
5	60	60	0
M	55	48	7
SD	13.23	16.81	5.70

# Dependent Samples $t$ Example (2)

1. Set alpha = .05
2. Null hypothesis:  $H_0: \mu_1 = \mu_2$ .  
Alternative is  $H_1: \mu_1 \neq \mu_2$ .
3. Calculate the test statistic:

$$SE_{diff} = \frac{SD}{\sqrt{n_{pairs}}} = \frac{5.70}{\sqrt{5}} = 2.55$$

$$t = \frac{\bar{D}}{SE_{diff}} = \frac{55 - 48}{2.55} = \frac{7}{2.55} = 2.75$$



# Dependent Samples t Example (3)

4. Determine the critical value of t.  
Alpha = .05, tails=2  
 $df = N(\text{pairs}) - 1 = 5 - 1 = 4.$   
Critical value is 2.776
5. Decision rule: is absolute value of sample value larger than critical value?
6. Conclusion. Not (quite) significant.  
Painfree does not have an effect.

# Using SPSS for dependent t-test

- Open SPSS
- Open file “SPSS Examples” (same as before)
- Go to:
  - “Analyze” then “Compare Means”
  - Choose “Paired samples t-test”
  - Choose the two IV conditions you are comparing. Put in “paired variables box.”

# Dependent t- SPSS output

**Paired Samples Statistics**

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PAINFREE	55.0000	5	13.2288	5.9161
	PLACEBO	48.0000	5	16.8077	7.5166

**Paired Samples Correlations**

		N	Correlation	Sig.
Pair 1	PAINFREE & PLACEBO	5	.956	.011

**Paired Samples Test**

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	PAINFREE - PLACEBO	7.0000	5.7009	2.5495	-7.86E-02	14.0786	2.746	4	.052



## Relationship between t Statistic and Power

- To increase power:

$$t = \frac{\bar{X} - \mu}{SE_M}$$

- Increase the difference between the means.
- Reduce the variance
- Increase N
- Increase  $\alpha$  from  $\alpha = .01$  to  $\alpha = .05$

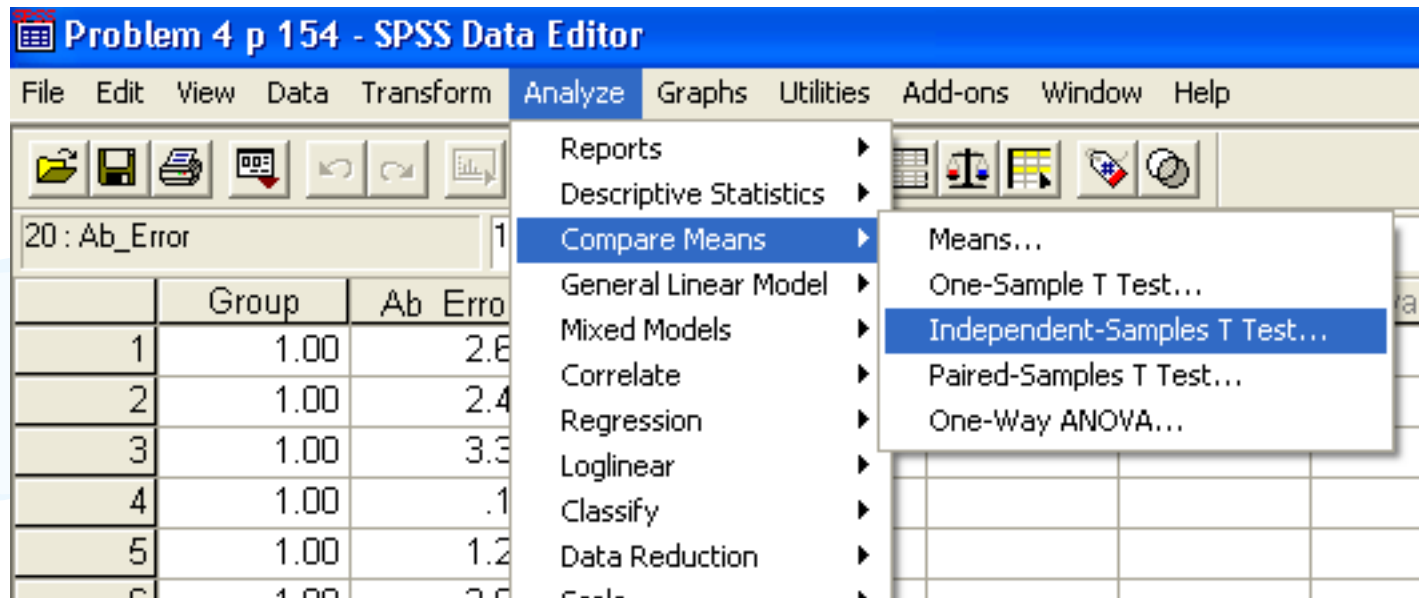




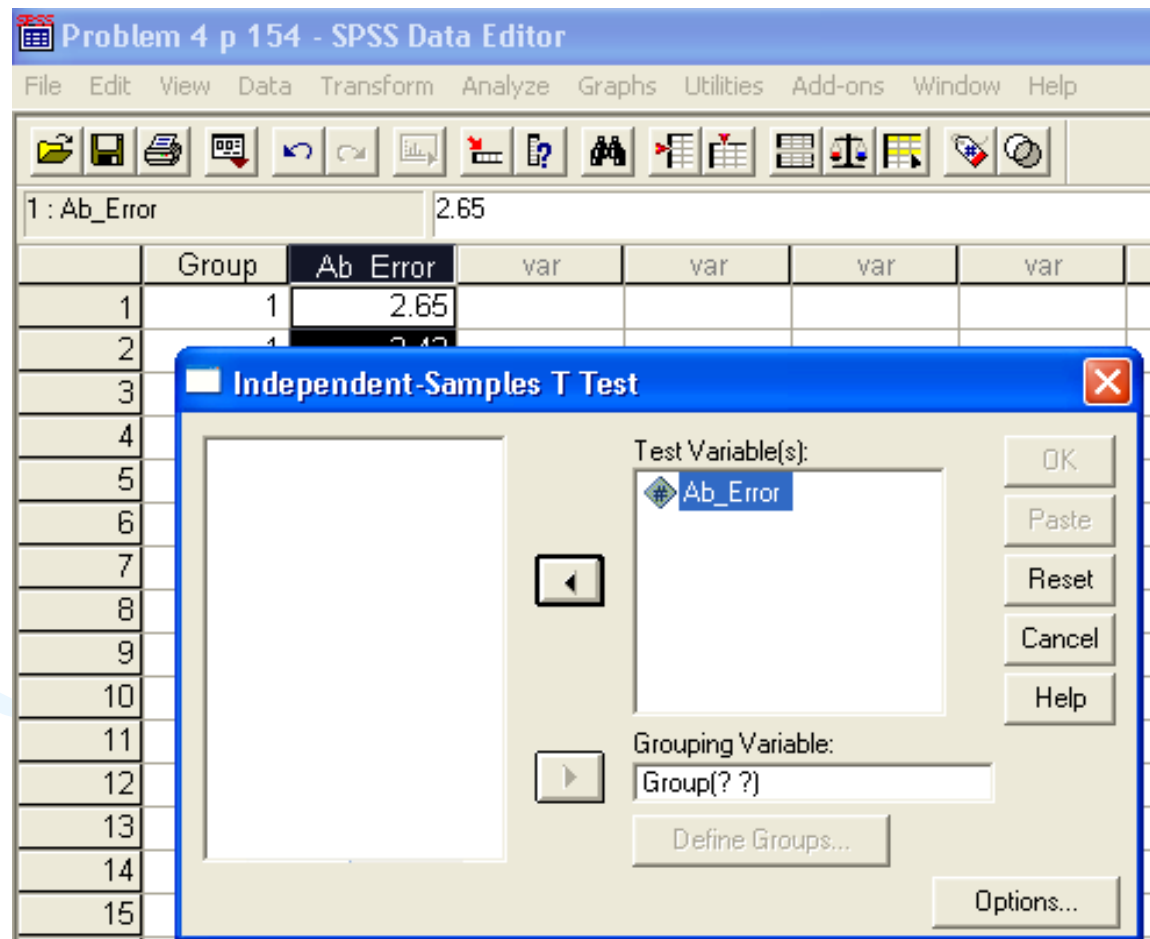
# To Increase Power

- Increase alpha, Power for  $\alpha = .10$  is greater than power for  $\alpha = .05$
- Increase the difference between means.
- Decrease the sd's of the groups.
- Increase N.

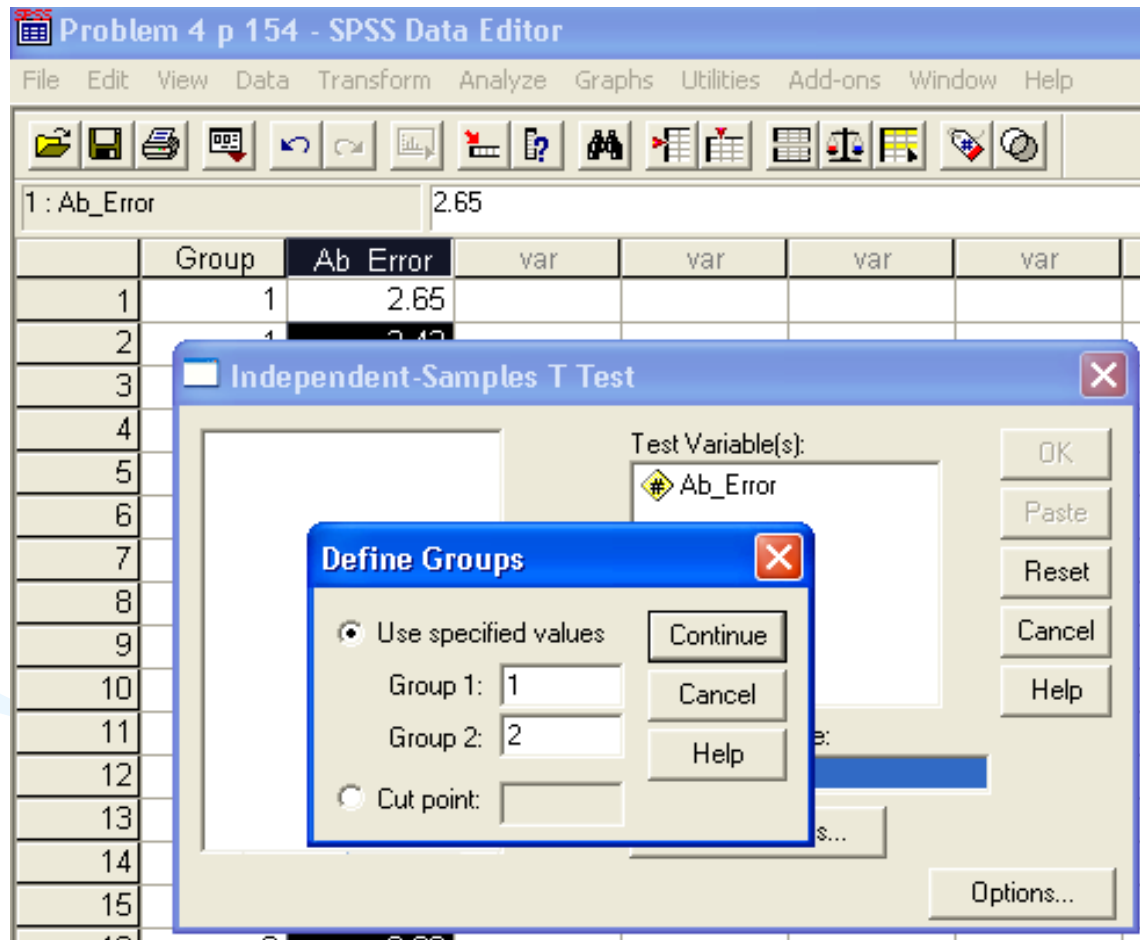
# Independent t-Test



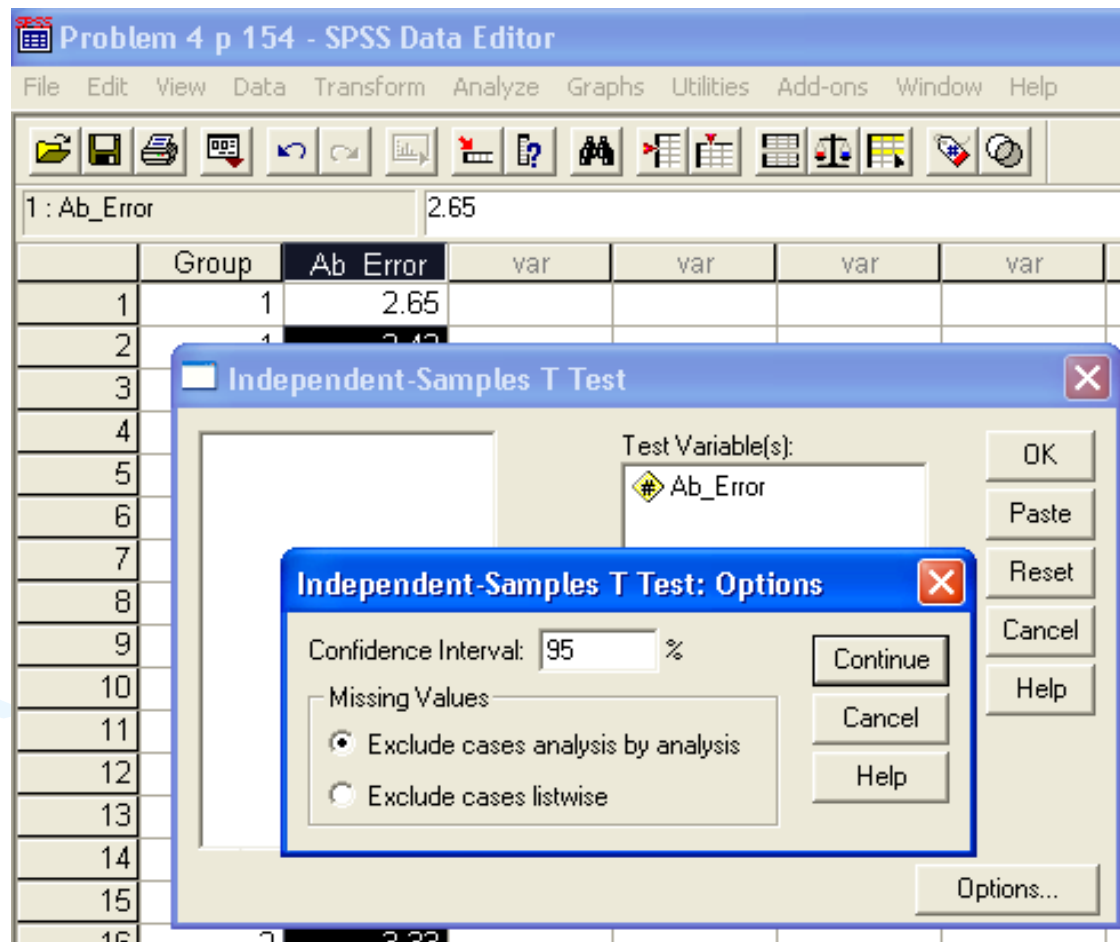
# Independent t-Test: Independent & Dependent Variables



# Independent t-Test: Define Groups



# Independent t-Test: Options



### Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
Ab_Error	Active	10	2.2820	1.24438	.39351
	Passive	10	1.9660	1.50606	.47626

## Independent t-Test: Output

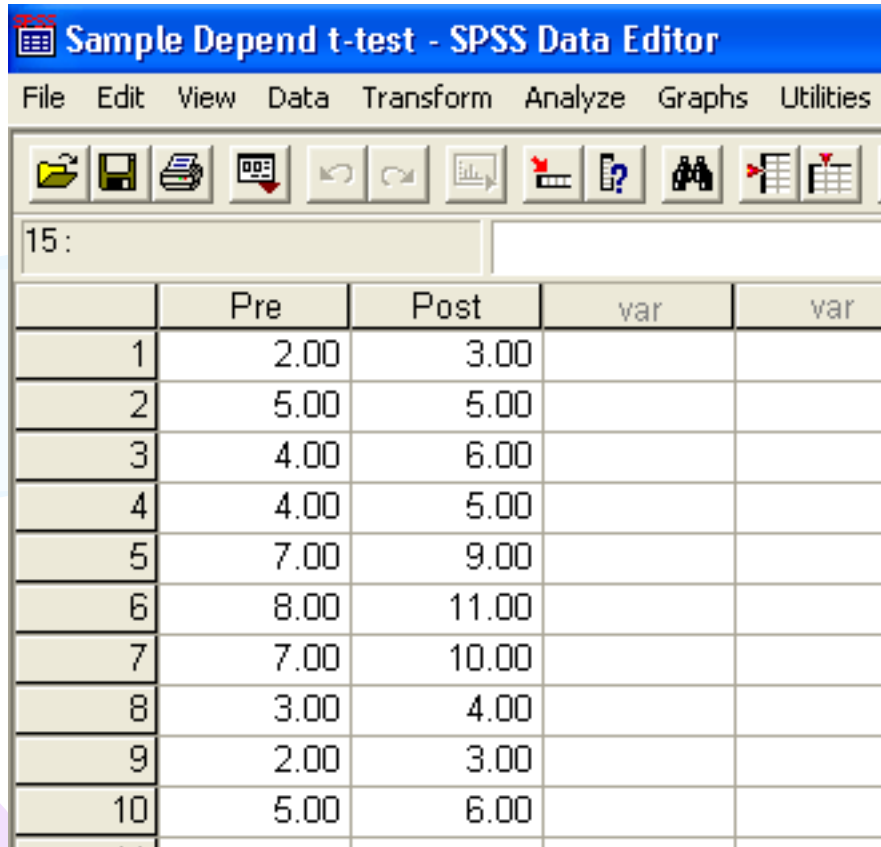
### Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
							Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)			Lower	Upper
Ab_Error	Equal variances assumed	.513	.483	.511	18	.615	.31600	.61780	-.98194	1.61394
	Equal variances not assumed			.511	17.382	.615	.31600	.61780	-.98526	1.61726

Assumptions: Groups have equal variance [ $F = .513$ ,  $p = .483$ , YOU DO NOT WANT THIS TO BE SIGNIFICANT. The groups have equal variance, you have not violated an assumption of t-statistic.

Are the groups different?  
  
 $t(18) = .511$ ,  $p = .615$   
 NO DIFFERENCE  
 2.28 is not different from 1.96

# Dependent or Paired t-Test: Define Variables



Sample Depend t-test - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities

15:

	Pre	Post	var	var
1	2.00	3.00		
2	5.00	5.00		
3	4.00	6.00		
4	4.00	5.00		
5	7.00	9.00		
6	8.00	11.00		
7	7.00	10.00		
8	3.00	4.00		
9	2.00	3.00		
10	5.00	6.00		

# Dependent or Paired t-Test: Select Paired-Samples

Sample Depend t-test - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help

15:

	Pre	Post
1	2.00	3.00
2	5.00	5.00
3	4.00	6.00
4	4.00	5.00
5	7.00	9.00
6	8.00	11.00
7	7.00	10.00
8	3.00	4.00
9	2.00	3.00
10	5.00	6.00

Analyze menu options:

- Reports
- Descriptive Statistics
- Compare Means
  - Means...
  - One-Sample T Test...
  - Independent-Samples T Test...
  - Paired-Samples T Test...**
  - One-Way ANOVA...
- General Linear Model
- Mixed Models
- Correlate
- Regression
- Loglinear
- Classify
- Data Reduction
- Scale
- Nonparametric Tests
- Survival
- Multiple Response



# Dependent or Paired t-Test: Select Variables

SPSS Sample Depend t-test - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help

15:

	Pre	Post
1	2.00	3.00
2	5.00	5.00
3	4.00	6.00
4	4.00	5.00
5	7.00	9.00
6	8.00	11.00
7	7.00	10.00
8	3.00	4.00
9	2.00	3.00
10	5.00	6.00
11		
12		

**Paired-Samples T Test**

Current Selections

Variable 1:

Variable 2:

Paired Variables:

Pre -- Post

OK Paste Reset Cancel Help Options...

# Dependent or Paired t-Test: Options

SPSS Sample Depend t-test - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help

15:

	Pre	Post
1	2.00	3.00
2	5.00	5.00
3	4.00	6.00
4	4.00	5.00
5	7.00	9.00
6	8.00	11.00
7	7.00	10.00
8	3.00	4.00
9	2.00	3.00
10	5.00	6.00
11		
12		

**Paired-Samples T Test**

Paired Variables:  
Pre -- Post

**Paired-Samples T Test: Options**

Confidence Interval: 95 %

Missing Values

- ☒ Exclude cases analysis by analysis
- ☐ Exclude cases listwise

Continue Cancel Help

Options...

### Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre	4.7000	10	2.11082	.66750
	Post	6.2000	10	2.85968	.90431

### Dependent or Paired t-Test: Output

### Paired Samples Correlations

	N	Correlation	Sig.
Pair 1 Pre & Post	10	.968	.000

### Paired Samples Test

		Paired Differences							
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Pre - Post	-1.50000	.97183	.30732	-2.19520	-.80480	t	df	Sig. (2-tailed)
							-4.881	9	.001

Is there a difference between pre & post?

$$t(9) = -4.881, p = .001$$

Yes, 4.7 is significantly different from 6.2